Understanding the Risk of COVID-19 Transmission on Trains

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Abstract

To understand the extent of the risk of the Covid-19 disease transmission among the train passengers, then there is a need for a formulation of an appropriate model that would adequately assess the risk of Covid-19 transmission. This work considered the passenger dynamic throughout their journey and explore how modelling approaches could be used to understand, thus mitigate possible transmission as passengers enter and leave a train.

Passengers Distribution

The passengers access to the platform from the midpoint of the bottom boundary and scatters as an exponential function.

\[ u_0 = e^{-(\beta_1(x-0.5)^2+\beta_2(y-0.8)^2)}, \]

where \( \beta_1 \) and \( \beta_2 \) characterise the level of initial spread.

Getting on the Train

Passengers motion towards the train carriage is model as in a fluid dynamics framework.

\[
\partial_t u + \nabla \cdot (\vec{v} u) = \nabla \cdot (\sigma \nabla u), \quad u \in C^1(\Omega)
\]

\( u \) is the crowd density, \( \vec{v} \) represents the velocity, \( \sigma \) is the diffusivity parameter. The initial domain \( \Omega \) is represented by the platform in Fig. 1.

More precisely, we simulate density of passengers movement with the convection-diffusion PDE and the passengers waiting on a railway platform for a train are initially scattered.

\[
X(t) = f(t) - \Delta t \frac{d}{dx} X(t)
\]

Using velocity based model, with function \( f \) which describes the behaviour and distance between passengers. \( X_{n+1} = f_{n+1}(X_n - X_{n+1}), f(r) = 0, r < d_1, f(r) = V, r > d_2, d_1, d_2 \) are social distance. Initially we will have everyone standing socially distant, \( r > d_1 \).

Simulation Result during Boarding

We compute the density of the passengers inside each door at each time step and see how it is affected by the parameter \( \beta_n \), which controls the scattering of the passengers on the platform.

Simulation Result for Departure

- A passenger has a waiting time before they can move.
- Takes a further travel time to leave the train.

Getting off the Train

- The queue has \( n \) passengers at location \( X_n(t), n = 0 \ldots N - 1 \).
- The queue move such that \( X_n \) exits before \( X_{n+1} \).
- When \( X_n = L \) then the \( n^{th} \) passenger will be assumed to have left the carriage with time \( \Delta_n \).
- If proportion \( \alpha \) of the passengers are infectious. The number of passengers encountered is then \( \alpha(L - X_n(0)) \).

Future Work

- Lifting passengers distribution restriction to simulate more realistic conditions.
- Exploring the simple viral load modelling of a queue system.
- Incorporating the equations of motion to better capture the characteristics of passenger population.

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